Section 11.3 Solutions

4. Let $f(x) = \frac{1}{x^5}$. Then f is continuous, positive, and decreasing on $[1, \infty)$, and

$$\int_1^\infty \frac{1}{x^5}\,dx = \lim_{t\to\infty}\left[-\frac{1}{4}x^{-4}\right]_1^t = \frac{1}{4}\lim_{t\to\infty}\left[1-\frac{1}{t^4}\right] = \frac{1}{4} < \infty$$

$\therefore \sum_{n=1}^\infty \frac{1}{n^5}$ $\boxed{\text{converges}}$ by the Integral Test.

6. Let $f(x) = \frac{1}{\sqrt{x+4}}$. Then f is continuous, positive, and decreasing on $[1,\infty)$, and

$$\int_1^\infty \frac{1}{\sqrt{x+4}}\,dx = \int_5^\infty \frac{1}{\sqrt{u}}\,du = \lim_{t\to\infty}\left[2\sqrt{u}\right]_5^t = 2\lim_{t\to\infty}\left[\sqrt{t}-\sqrt{5}\right]$$

$= \infty$. Therefore, $\sum_{n=1}^\infty \frac{1}{\sqrt{n+4}}$ $\boxed{\text{diverges}}$ by the Integral Test.

Section 11.5 Solutions

2. $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \cdots = 2\sum_{n=1}^\infty \frac{(-1)^{n-1}}{2n+1}$, where $b_n = \frac{1}{2n+1}$. Clearly, $\{b_n\}$ is a decreasing sequence, ie., $b_{n+1} < b_n$, and $\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{2n+1} = 0$, so $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \cdots$ $\boxed{\text{converges}}$ by the Alternating Series Test (AST)

4. $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \cdots = \sum_{n=2}^\infty \frac{(-1)^n}{\sqrt{n}}$, where $b_n = \frac{1}{\sqrt{n}}$. Clearly, $b_{n+1} = \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}} = b_n$ and $\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$, so $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \cdots$ $\boxed{\text{converges}}$ by the AST

6. $b_n = \frac{1}{\ln(n+4)}$ is decreasing (because $f(x) = \ln x$ is an increasing function) and $\lim_{n\to\infty}\frac{1}{\ln(n+4)} = 0$, so $\sum_{n=1}^\infty \frac{(-1)^{n-1}}{\ln(n+4)}$ $\boxed{\text{converges}}$ by AST

14. $b_n = \arctan n \to \frac{\pi}{2}$ as $n \to \infty$, so $\sum_{n=1}^\infty (-1)^{n-1}\arctan n$ <u>diverges</u> by the Divergence Test.